A Method of Representing Fan-Wing Combinations for Three-Dimensional Potential Flow Solutions

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A method of representing fan-wing combinations for three-dimensional potential flow solutions has been developed to determine the theoretical aerodynamic characteristics of vertical/short takeoff and landing (V/STOL) type aircraft. The approach is to represent the wing with a network of horseshoe vortices distributed on the surface and trailing behind. The jet efflux is represented with a rectangular horseshoe vortex network in the shape of a curved tube. Flow is induced through the tube by flaring out its end, which is several chord lengths from the wing. The strength of each vortex segment is determined by requiring that the flow be parallel to the surface at a number of control points. The equations resolve into a series of *N* equations with *N* unknowns that may be solved by a digital computer using standard matrix inversion methods. The coefficients of the equations are functions of geometry and freestream velocity direction only. The force on each vortex segment is then computed as the cross product of the local velocity vector and the vortex strength. Pressure coefficients of the control points are computed by considering the vortex segments as distributed vorticity in both the spanwise and chordwise directions. Extremely good correlation of computed data with test data indicates that the method described is a significant contribution to the advancement of aerodynamics, and it appears to have a wide application.

Nomenclature

a = half-side of square, ft
a, b = represent control points
c = radius of circle, ft
C_L = lift coefficient
C_P = pressure coefficient
D = fan diameter, ft
$\mathbf{F}$ = resultant force vector on a vortex segment
G = influence coefficient
l = length of a vortex segment
$\mathbf{n}_a$ = unit vector normal to surface at control point a
r = radius of a point, ft
$\mathbf{v}$ = velocity vector, fps
$\mathbf{v}_a$ = resultant induced velocity vector at control point a
V = velocity function column matrix
$\mathbf{V}_0$ = freestream velocity vector, fps
V_j = jet efflux velocity, fps
ΔV = velocity induced by a vortex sheet, fps
x, y, z = coordinate axes, ft
α = angle of attack, deg
δ_f = flap deflection angle, deg
Γ = circulation or vortex strength, ft²/sec
γ = vorticity, fps
θ = jet flow tube flare angle, deg
ρ = mass density of fluid, slugs/ft³
ψ = angle of yaw, deg; or stream function, ft²/sec
1, 2, 3, 4 = different vortex segments

Introduction

WITH the increasing interest in airplanes that will take off and land either vertically or in a very short distance (V/STOL), new questions are being generated that need to be answered. The specific area of concern for this paper is the interaction between a lift fan and wing that are in close proximity to one another. This combination cannot be analyzed properly by simple methods. The present technology provides us with two-dimensional answers[1] and for three-dimensional simple shapes,[2] but three-dimensional solutions for complex shapes including influences of the jet efflux vorticity on the freestream air are not available. Model testing of a fan-wing combination is quite difficult and expensive to accomplish properly, and large velocity perturbations cause questionable wind-tunnel corrections, even under the best test conditions.

Since flow induced by a propulsion fan, near or in a wing, can have a pronounced effect on the configuration, a study has been initiated to find the effects of these induced flows on a theoretical basis. The objectives of this paper are to describe a method of representing fan-wing combinations for potential flow calculations and to compare the theoretical calculations with test data.

Theoretical Considerations

General

The method of approach taken is to represent the fan-wing configuration by a three-dimensional vortex lattice network. A nonplanar wing can be represented by a surface network of horseshoe vortices whose bound portions are coincident with constant percent chord lines and whose trailing portions are restrained to the cambered wing or flap surface back to the trailing edge. Aft of the trailing edge, the vortices become free and parallel to the local flow. This nonplanar capability is an advancement over the well-used theory that forced the trailing vorticity to be parallel to the freestream flow. This mathematical model is shown in Fig. 1.

The jet efflux from the lifting fan is represented by a curved pipe or flow tube protruding from the lower side of the wing.

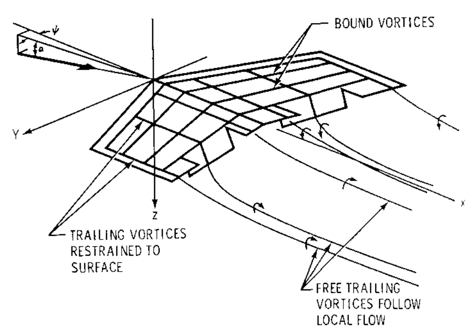

Fig. 1 Mathematical model of wing.

Presented as Preprint 65-85 at the AIAA 2nd Aerospace Sciences Meeting, New York, N. Y., January 25–27, 1965; revision received May 20, 1965.

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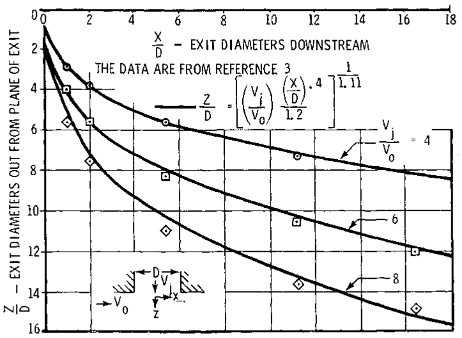


Fig. 2 Effect of jet exit velocity on wake trajectory.

For simplifying reasons the tube has a square cross section whose area is equal to the area of the fan exit that it is to represent. The tube is essentially a surface vortex network that has been folded so that its edges meet, forming a tube.

In order to properly represent the efflux of the fan, it was necessary to arrive at some empirical method of calculating its trajectory. Reference 3 presents some test data on the trajectory of the lines of maximum total head from a jet directed normal to the wind. The data are shown in Fig. 2. To use the trajectory data at other values of V_j/V_0, an equation was derived from it. The simplest equation that best fits the data is

$$Z/D = [(V_j/V_0)(X/D)^{.4}/1.2]^{1/1.11}$$

To determine the feasibility of this method of flow tube representation, an isolated tube that was shaped for a jet velocity ratio V_j/V_0 of five was programed for the computer in a manner similar to that shown in Fig. 3. The last set of tube surfaces were flared so as to suck fluid down through the tube. Calculations were made at several different flare angles. The velocity ratio was determined from the average pressure coefficient inside the flow tube with the following equation:

$$V_j/V_0 = [1 - C_{P_{av}}]^{1/2}$$

The resulting data are shown in Fig. 4 and follow the expected trend of increasing flow with increasing tube flare. Since the tube was shaped for a velocity ratio of five, one would pick a flare angle of 16.5° to properly represent the tube. Similar calculations must be done to match the tube shape and flare angle to represent other velocity ratios.

Circle-Square Similarity

The flow tube was represented with a square cross section rather than a circular one because of simplicity and the desire to use a program currently available. Fig. 5 compares the two-dimensional streamlines for a circle and square and indicates only small differences at distances greater than one diameter from the center.

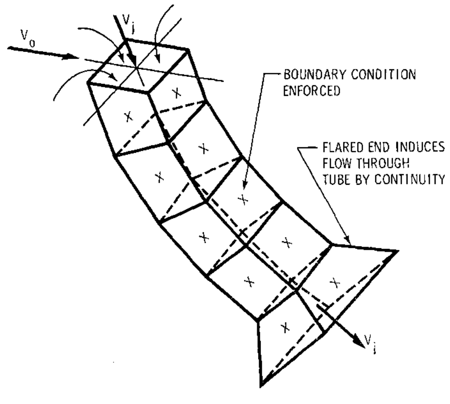


Fig. 3 Jet efflux flow tube.

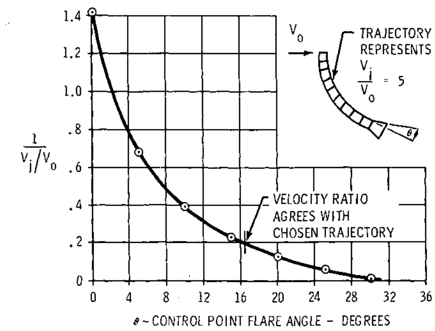


Fig. 4 Calibration of isolated flow tube.

The stream function for a circle in potential flow from Ref. 4 is

$$\psi_C = -V_0 y[1 - (c^2/r^2]$$

and that for a vortex is

$$\psi_V = -V_0 y - (\Gamma/2\pi) \ln r$$

By considering a square with a vortex at each corner and requiring that the flow be zero at the center of the front and back faces and parallel at the center of the top and bottom faces, one arrives at the resulting stream function for the square:

$$\psi_S = -V_0[y + (a/2.4) \ln(r_1 r_2/r_3 r_4)]$$

The circulation strength of all four vortices is equal in magnitude, but the two image vortices are opposite in direction from the two upper ones.

Solution of Vortex Strengths

After the geometry of the problem is properly defined and the vortex network is established, it is necessary to determine the strength of all of the individual vortices. The basic method is to enforce the requirement that the normal component of the total velocity at each specified control point is zero. The control points are placed at an equal distance from the one bound and two trailing vortices of each horseshoe. This provides an unknown vortex strength and a known boundary condition for each of the horseshoes in the network.

Now in order to enforce our boundary condition that the total velocity normal to the surface at the control point a must be zero, we can say that the dot product of the unit normal vector at a and the resultant velocity must be zero, or

$$\mathbf{n}_a \cdot (\mathbf{v}_a + \mathbf{V}_0) = 0$$

or

$$\mathbf{n}_a \cdot \mathbf{v}_a = -\mathbf{n}_a \cdot \mathbf{V}_0$$

The left-hand side of the preceding equation is the product of a matrix depending only on the geometry of the network,

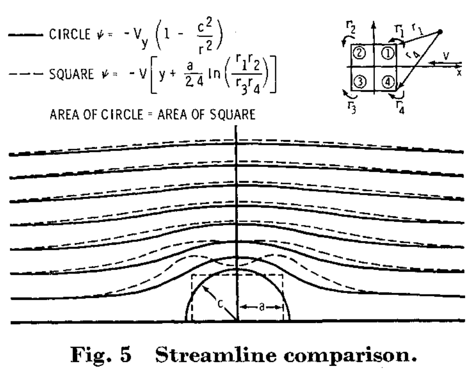


Fig. 5 Streamline comparison.

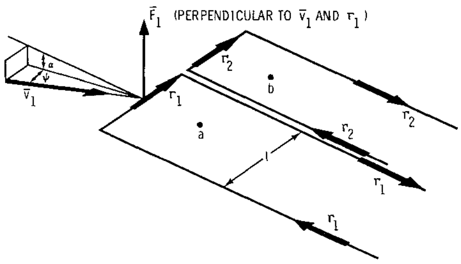


Fig. 6 Two horseshoe network.

and the right-hand side is a function of the boundary point normals and the freestream velocity vector, so that we may rewrite it in the following matrix form:

$$\{V\} = -[G]\{\Gamma\}$$

or solving for the vortex strengths:

$$\{\Gamma\} = -[G]^{-1}\{V\}$$

Calculation of Forces and Moments

After solving for the individual vortex strength, we must now calculate the forces generated by those vortices that are contained in the represented surface. We can modify our two-dimensional Kutta-Joukowski Law to fit a three-dimensional case by considering the network in Fig. 6. We see that the force on a constrained vortex segment is the cross product of the velocity and circulation times the density and length of the segment, or

$$\mathbf{F} = \rho l_1(\mathbf{v}_1 \times \Gamma_1)$$

where the velocity is the vector sum of all of the induced velocities and the freestream velocity at the midpoint of the vortex segment being considered. One exception here is that the strength of the vortex being considered cannot be included when determining the resultant velocity, since a straight vortex has no effect on itself.

When forces on the chordwise segments are considered, all of the vortex strengths that are coincident must be algebraically subtracted to get the net vortex strength at that point. For example, if Γ_1 and Γ_2 were equal, the net strength for the chordwise segment between them would be zero if they were the only two existing in that segment. The previous force equation can be resolved into its component parts in the lift, drag, and side force directions; then the force coefficients for the surface will be the sum of all of the individual coefficients for each segment.

The moment coefficients with respect to a given point are the sum of all of the individual coefficients times their appropriate moment arms.

Calculation of Pressure Coefficient

The surface of a wing may be considered to consist of a sheet of vorticity γ where

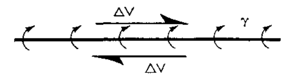
$$\gamma = d\Gamma/dl$$

This vortex sheet is composed of two separate vortex sheets that are superimposed. One sheet is that due to considering the spanwise vortex segments as distributed in a sheet, and the other is that due to considering the chordwise segments as distributed in a sheet. One sheet is thus composed of vorticity in a spanwise direction, and the other is composed of vorticity in a chordwise direction.

If we assign a length l to each segment, then the vorticity at that segment will be

$$\gamma = \Gamma/l$$

Fig. 7 Velocity induced by a flat sheet of vorticity.



Here, again, in dealing with the chordwise segments where several vortices are superimposed, we must properly account for the net strength at that point in the same manner that we did for figuring the forces. To find the values of γ at the control point, we can average the values of chordwise vorticity ahead and behind. For the spanwise vorticity we average those values on each side.

From the vortex sheet theory of Ref. 5, the effect of a flat sheet of vorticity is to induce a discontinuous component of velocity at its surface as shown in Fig. 7. The magnitude of the velocity is

$$\Delta V = \gamma/2$$

and it is parallel to the surface at the surface and perpendicular to the axes of the vortices used to calculate it.

The spanwise and chordwise components of the vorticity-induced velocity must be added to those induced by the vortex network and the freestream velocity in order to compute the velocity on the upper and lower sides of the surface. The pressure coefficient is then computed from the resultant velocities at the control point:

$$C_P = 1 - (v/V_0)^2$$

The Boeing Company has a program in operation that performs the calculations and "bookkeeping" chores that have been described in this section. The program is reported in Ref. 6 and has been used to study such things as flap-span and chord effects, ground effect, and wind-tunnel wall corrections.

Data Correlation

In order that the method of representation could be substantiated, some computations were made using a fan-in-wing configuration that had been tested in the 40 × 80-ft wind tunnel at the NASA Ames Aeronautical Lab.. The test is reported in Ref. 7.

Figure 8 shows the wing planform and the layout of the vortex segments. The wing consisted of 48 control points. The flow tube was represented with 40 control points, making a total of 88 control points for the symmetrical configuration. The computer program has the capability to handle 100 control points. Figure 9 shows a two-view of the two networks combined and provides a three-dimensional description of the fan-wing system.

As a matter of interest, this system took approximately 21 minutes of IBM 7090 time for each run. A run consisted of one set of geometry, one angle of attack and yaw, and one jet velocity ratio.

In analyzing the computed data for the fan-wing configuration, only those components from the segments in the surface of the wing were considered. The segments of the flow tube were used only to induce the flow and provide vorticity, so

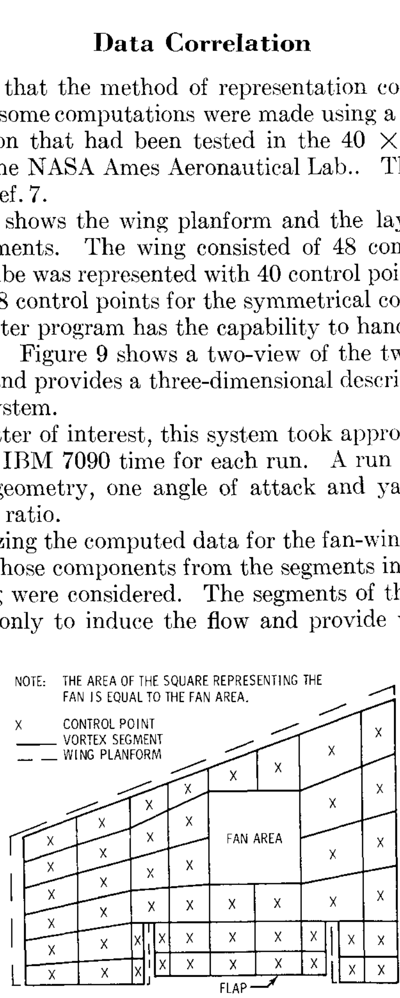


Fig. 8 Layout of vortex network for wing.

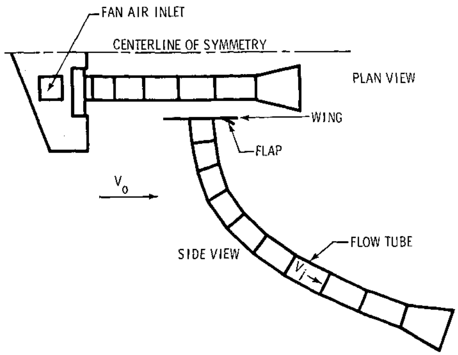


Fig. 9 Combined fan-wing system.

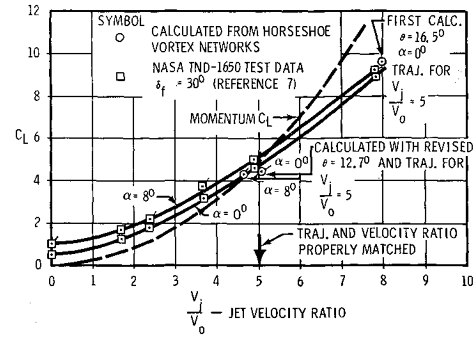


Fig. 10 Fan-in-wing data comparisons.

that they should contribute no forces to the configuration. However, the four segments at the top of the tube that are coincident with the wing surface are an exception.

The results are shown in Fig. 10. Runs were made at both 0° and 8° angle of attack, and both indicate extremely good correlation for this "first-cut" approach.

The first calculation was made using the flare angle of 16.5° as determined by the isolated flow tube calibration mentioned earlier. This resulted in a jet velocity that was too large and produced a C_L that was higher than the test data indicated. The reason for the difference in velocity ratio is that the jet trajectory determined from an isolated flow tube may be wrong when under the influence of a wing. The run was recalculated using a new flare angle, which resulted in a much closer jet velocity simulation, but the C_L dropped below the test data indicating that the jet trajectory could possibly be causing the difference. It would be of interest to compute the momentum balance of the tube to determine if there is any momentum violation. This is planned to be done in the future, perhaps even to the point of allowing the momentum considerations to shape the tube trajectory.

The increment between the two angles of attack does appear to be consistent with the test data. The lift due to the momentum change of the fan air (momentum C_L) is also shown for comparison. The difference between the data points and the momentum lift is a measure of the interference of the wing on the fan performance.

Induced drag data are also available from the program but have not been analyzed as yet. Also, pitching moments can be obtained by using the selected vortex segments multiplied by their individual moment arms.

Concluding Remarks

It is felt that the three-dimensional representation of the fan-wing configuration that includes jet efflux vorticity is a significant contribution to the advancement of aerodynamics and appears to have a wide application. It is desired to use this method in studying the effects on the aerodynamic characteristics of wings in the vicinity of lifting propulsion devices and on stability and control, ground effects, and wind-tunnel corrections. It is also desired to refine the method of determining the flow tube trajectory to include a momentum balance concept.

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